

LINEAR CIRCUIT ANALYSIS |
(EED) – U.E.T. TAXILA |
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05

INTRODUCTION

After learning the basic laws and theorems for circuit analysis, an active element of paramount importance may be studied.

This active element is known as operational amplifier, or op amp for short.

The op amp is a versatile circuit building block.

The Op Amp is an electronic unit that behaves like a voltage-controlled voltage source.

It may also be current-controlled current source.

INTRODUCTION

An op amp can sum signals, amplify a signal, integrate it, or differentiate it.

The ability of the op amp to perform these mathematical operations is the reason it is called operational amplifier.

The op amps are extensively used in analog circuit design.

Op amps are popular in practical circuit designs because they are versatile, inexpensive and easy to use.

OPERATIONAL AMPLIFIERS

An Op Amp is an active element designed to performs mathematical operations of addition, subtraction, multiplication, division, integration and differentiation.

The op amp is an electronic device consisting of a complex arrangement of resistors, capacitors, transistors and diodes.

A full discussion of what is inside the op amp is beyond the scope of this book.

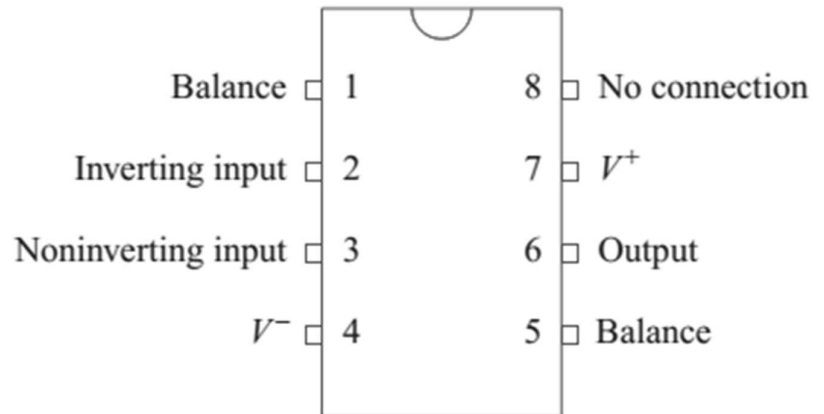
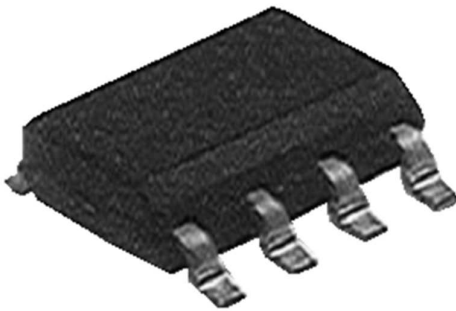
Here op amp is studied as an active circuit element and what takes place at its terminals.

OPERATIONAL AMPLIFIERS

Op amps are commercially available in integrated circuit packages in several forms.

A typical one is the eight-pin dual-in-line package (DIP).

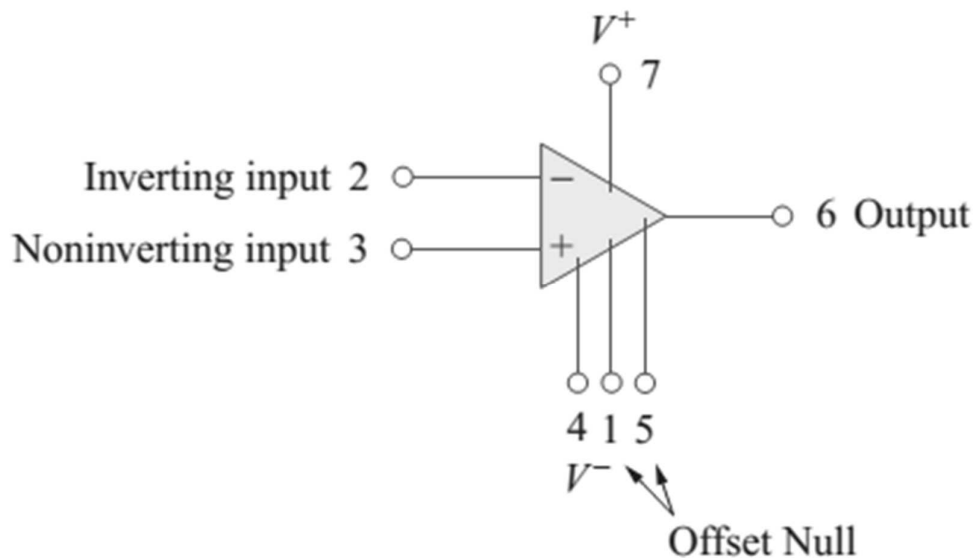
Pin configuration;



OPERATIONAL AMPLIFIERS

The circuit symbol and description of pins, is shown;

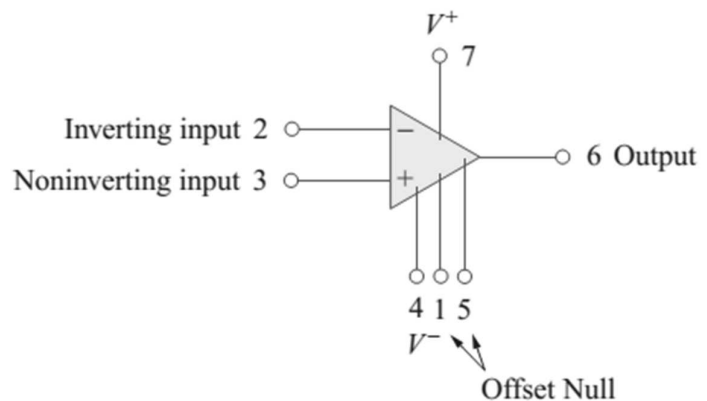
Op amp has two inputs (*pin 2, 3*) and one output (*pin 6*).



OPERATIONAL AMPLIFIERS

The inputs are marked with minus (-) and plus (+) to specify inverting and non-inverting inputs, respectively.

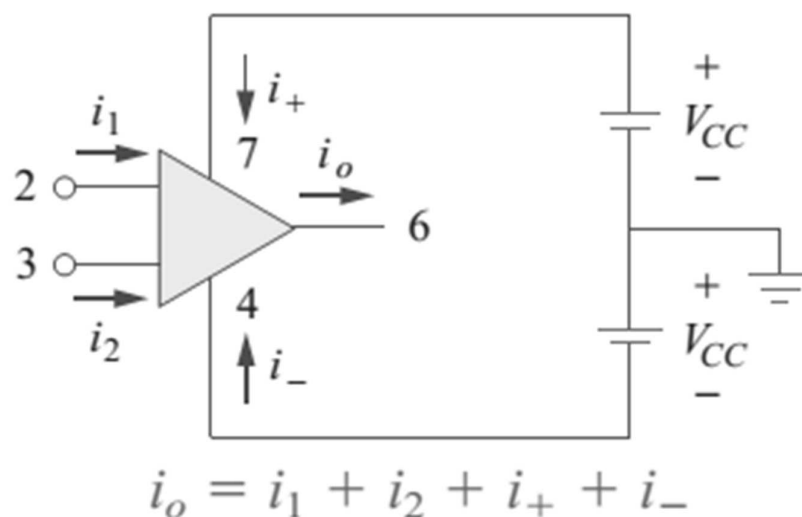
An input applied to the non-inverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.



OPERATIONAL AMPLIFIERS

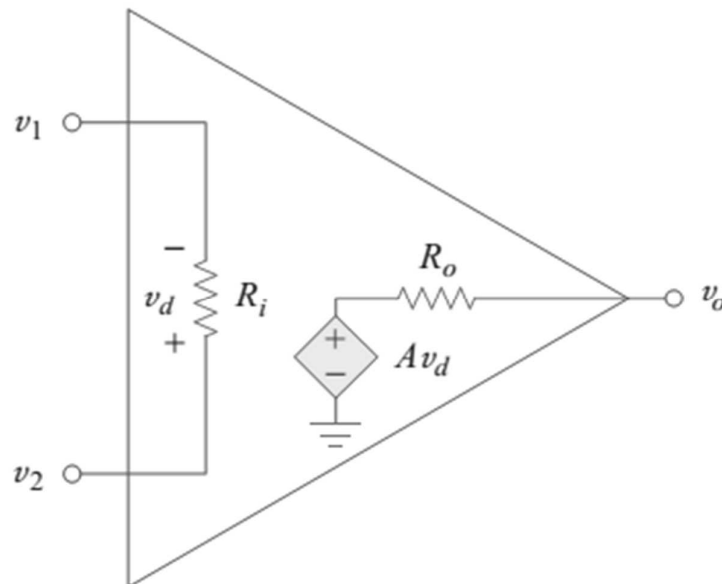
As an active element, the op amp must be powered by a voltage source.

Most often, power supplies are ignored for simplicity.



OPERATIONAL AMPLIFIERS

The equivalent circuit model of op amp is shown;



OPERATIONAL AMPLIFIERS

The output section consists of a voltage-controlled voltage source in series with output resistance R_o .

The differential input voltage; $v_d = v_2 - v_1$

Where v_1 is the voltage between the inverting terminal and ground and v_2 is the voltage between non-inverting terminal and ground.

Op amp senses the voltage difference between two inputs, multiplies it by the gain A and outputs the voltage as;

$$v_o = Av_d = A(v_2 - v_1)$$

OPERATIONAL AMPLIFIERS

The parameter A is called Open-Loop Gain because it is the gain of the op amp without any external feedback from output to input.

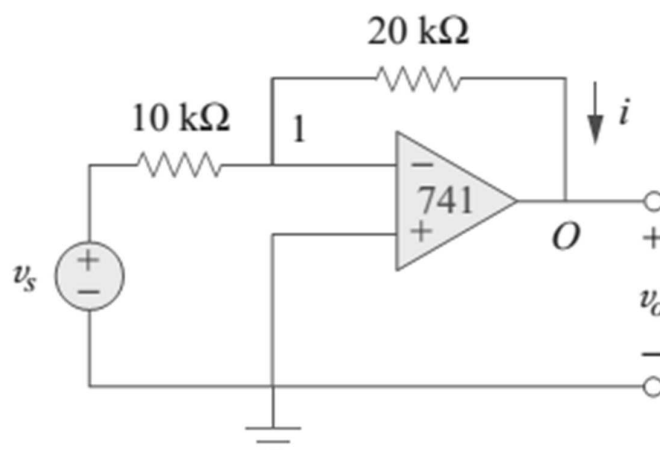
Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to $10^{13} \Omega$	$\infty \Omega$
Output resistance, R_o	10 to 100Ω	0Ω
Supply voltage, V_{CC}	5 to 24 V	

OPERATIONAL AMPLIFIERS

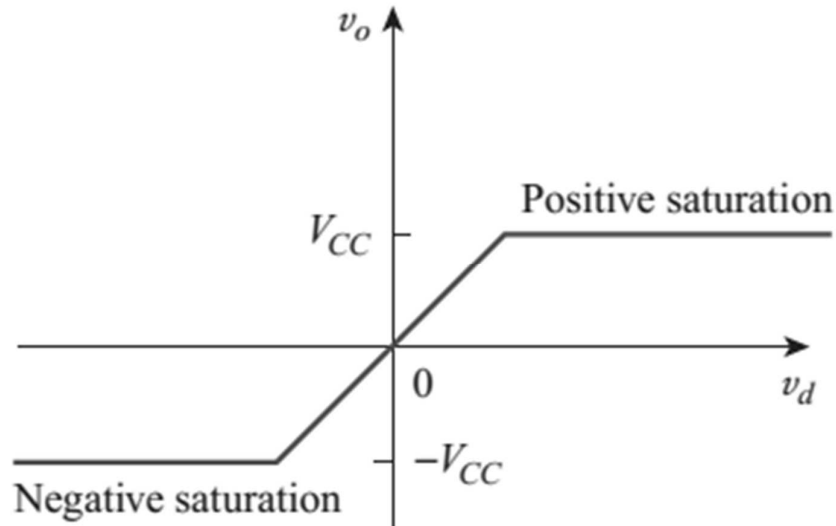
A negative feedback is achieved when output is fed back to the inverting terminal of the op amp.

In case of negative feedback, the ratio of the output voltage to the input voltage is called Closed-Loop Gain.



OPERATIONAL AMPLIFIERS

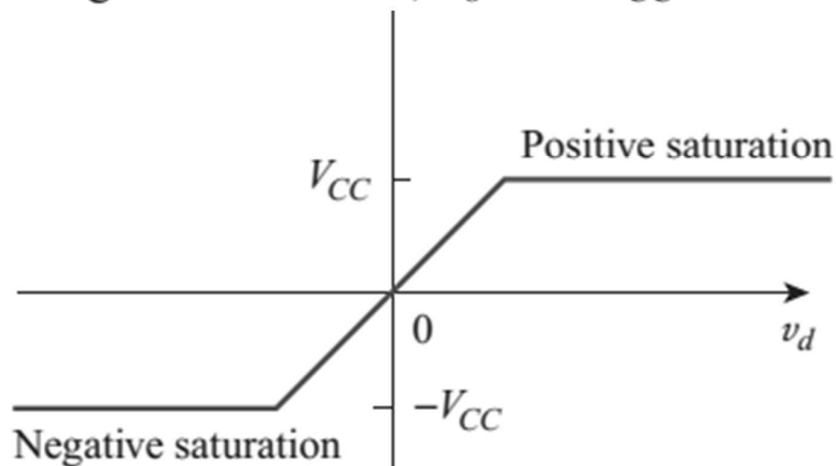
A practical limitation of the op amp is that magnitude of its output voltage cannot exceed the battery voltage, $|V_{CC}|$.



OPERATIONAL AMPLIFIERS

There are three regions of operation for op amp.

1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \leq v_o = Av_d \leq V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.



OPERATIONAL AMPLIFIERS

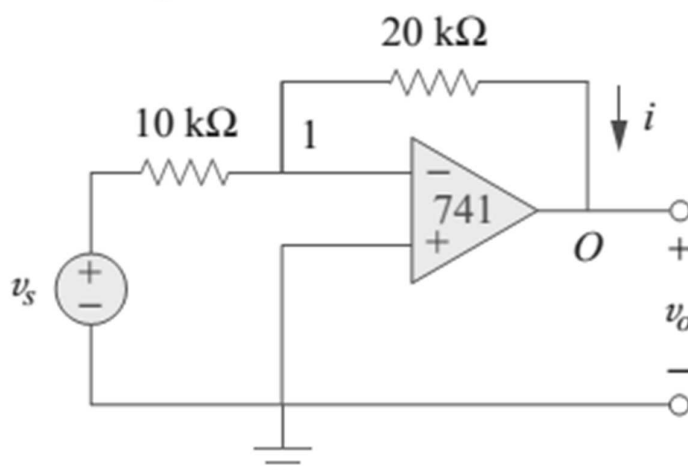
For linear circuit analysis, it is assumed that op amps operate in the linear mode throughout the book.

It means that output voltage of op amp is restricted as following;

$$-V_{CC} \leq v_o \leq V_{CC}$$

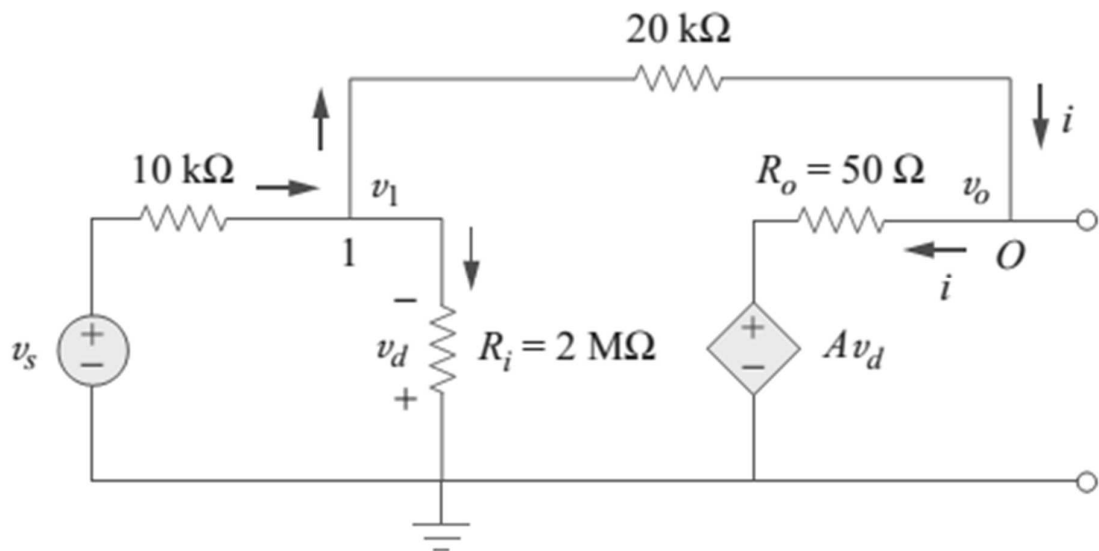
PROBLEMS

A 741 op amp has an open-loop voltage gain of 2×10^5 , input resistance of $2 \text{ M}\Omega$, and output resistance of 50Ω . The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s . Determine current i when $v_s = 2 \text{ V}$.



(-1.99, 0.199A)

PROBLEMS



IDEAL OP AMP

An Ideal Op Amp is an amplifier with infinite open-loop gain, infinite input resistance and zero output resistance.

Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to $10^{13} \Omega$	$\infty \Omega$
Output resistance, R_o	10 to 100Ω	0Ω
Supply voltage, V_{CC}	5 to 24 V	

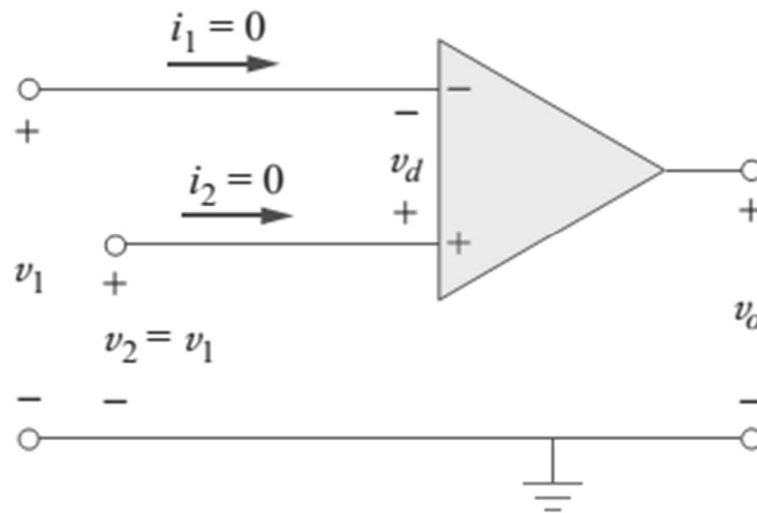
IDEAL OP AMP

The equivalent circuit of ideal op amp is shown;

$$i_1 = 0, \quad i_2 = 0$$

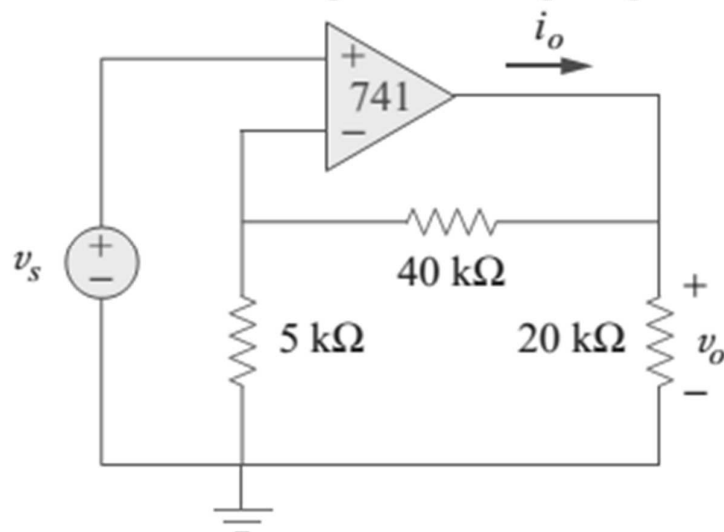
$$v_d = v_2 - v_1 = 0$$

$$v_1 = v_2$$



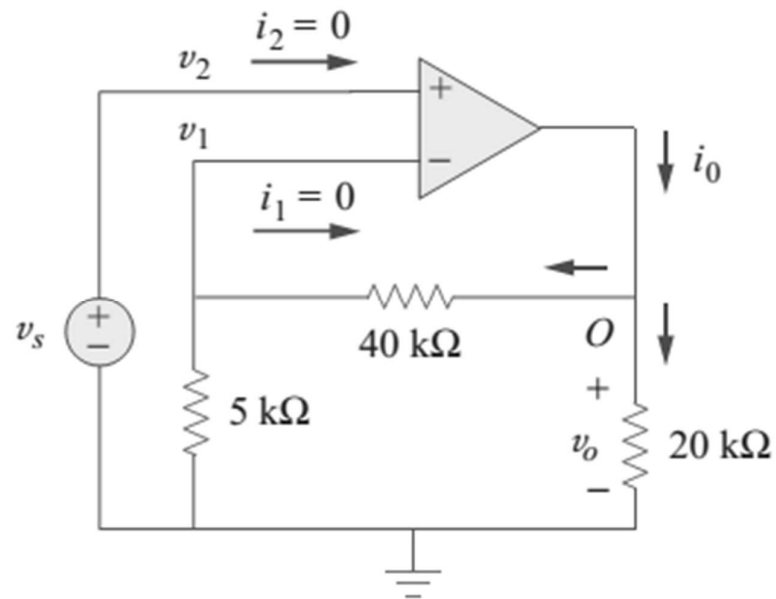
PROBLEMS

Rework Practice Prob. 5.1 using the ideal op amp model.



(9, 0.65mA)

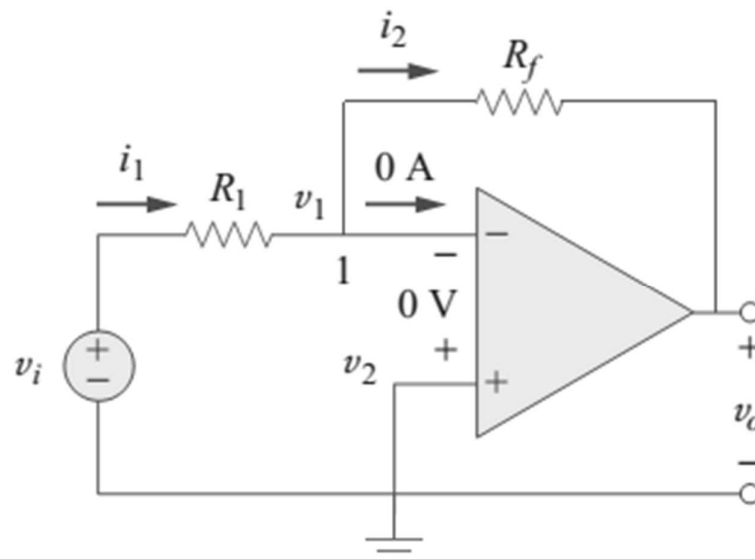
PROBLEMS



(9, 0.65mA)

INVERTING AMPLIFIER

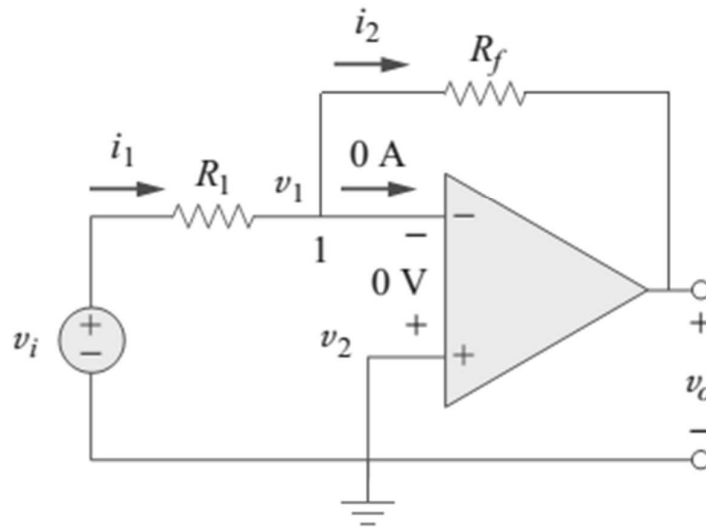
An Inverting Amplifier reverses the polarity of the input signal while amplifying it.



INVERTING AMPLIFIER

Applying KCL at node 1;

$$i_1 = i_2 \Rightarrow \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$



INVERTING AMPLIFIER

$$i_1 = i_2 \Rightarrow \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

For ideal op amps;

$$v_1 = v_2 = 0$$

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f} \quad v_o = -\frac{R_f}{R_1}v_i$$

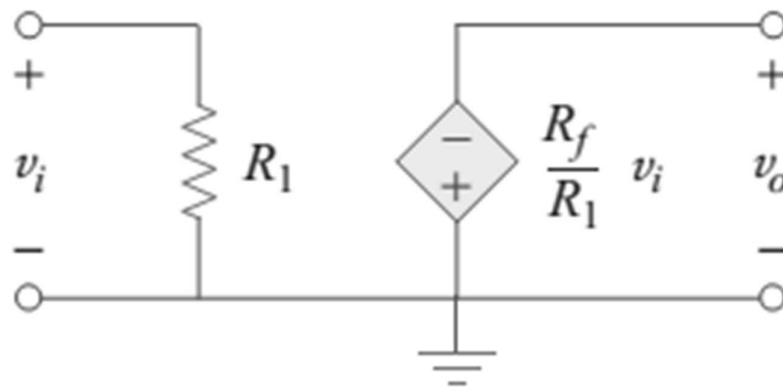
Voltage gain;

$$A_v = v_o/v_i = -R_f/R_1$$

INVERTING AMPLIFIER

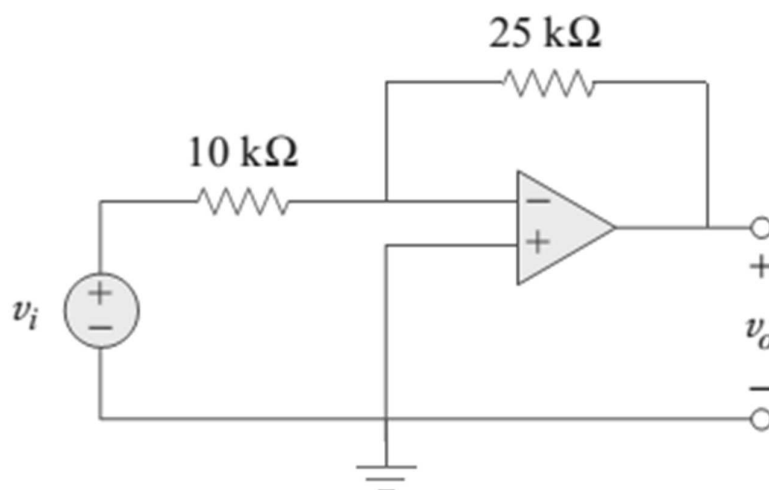
Voltage gain only depends on the external elements connected to the op amp.

Equivalent circuit;



PROBLEMS

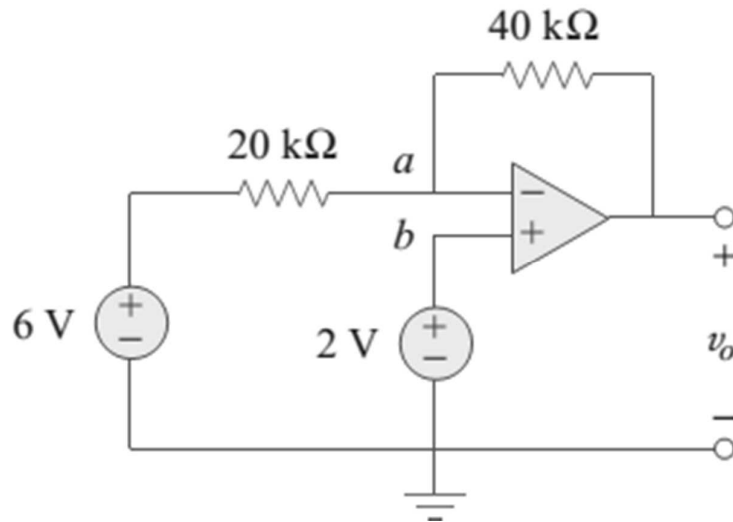
Refer to the op amp in Fig. 5.12. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o , and (b) the current in the 10-k Ω resistor.



(-1.25V, 50 μ A)

PROBLEMS

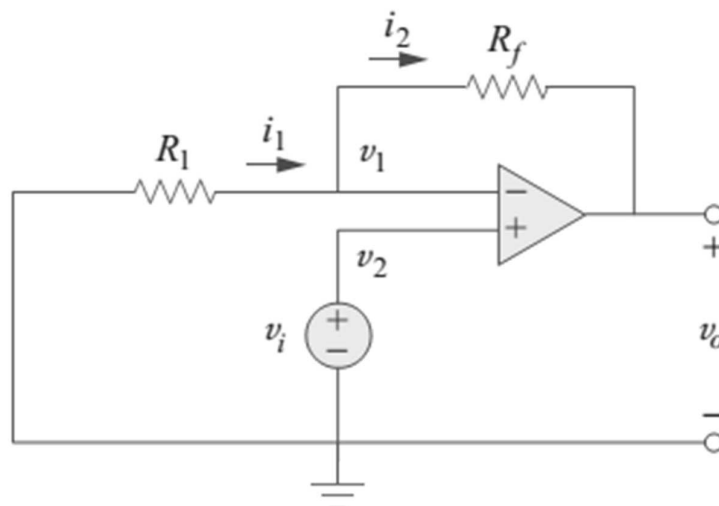
Find the output voltage?



(-6V)

NON-INVERTING AMPLIFIER

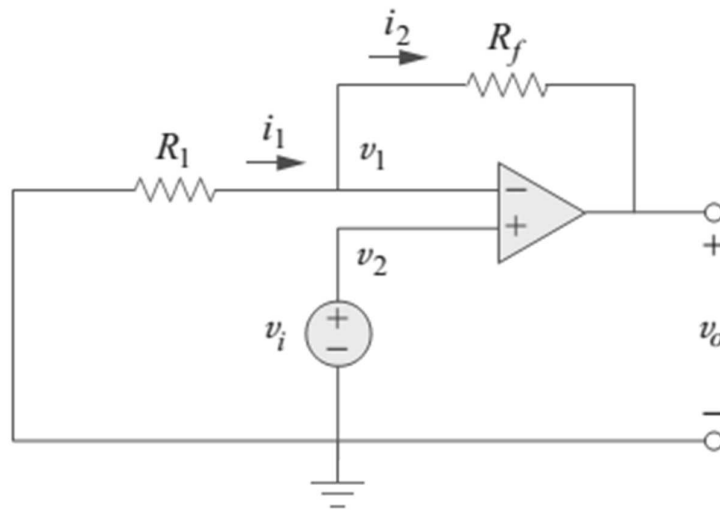
In this amplifier, the input is directly applied at non-inverting input terminal.



NON-INVERTING AMPLIFIER

Applying KCL at node 1;

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$



NON-INVERTING AMPLIFIER

$$i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

For ideal op amp;

$$v_1 = v_2 = v_i$$

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f} \quad v_o = \left(1 + \frac{R_f}{R_1}\right)v_i$$

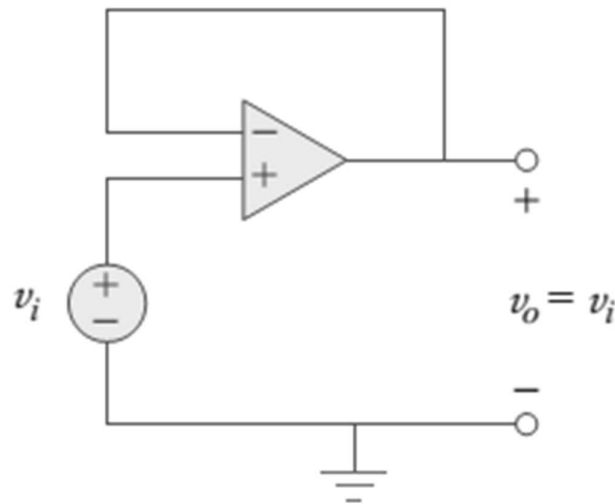
Voltage gain;

$$A_v = v_o/v_i = 1 + R_f/R_1$$

NON-INVERTING AMPLIFIER

A Non-inverting Amplifier is an op amp circuit designed to provide a positive voltage gain.

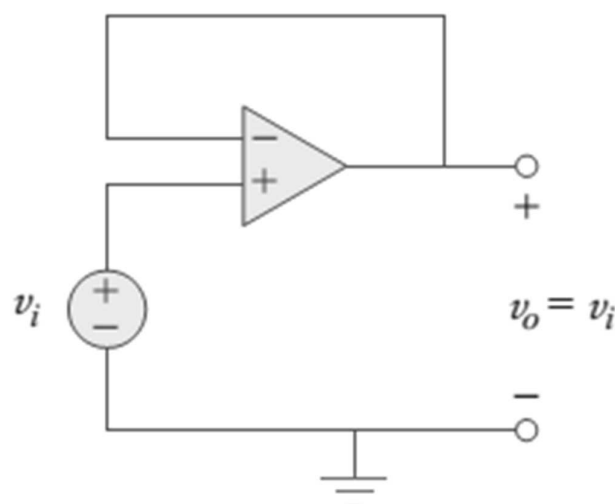
If $R_f=0$, $R_I=\infty$ or both, the gain becomes 1.



NON-INVERTING AMPLIFIER

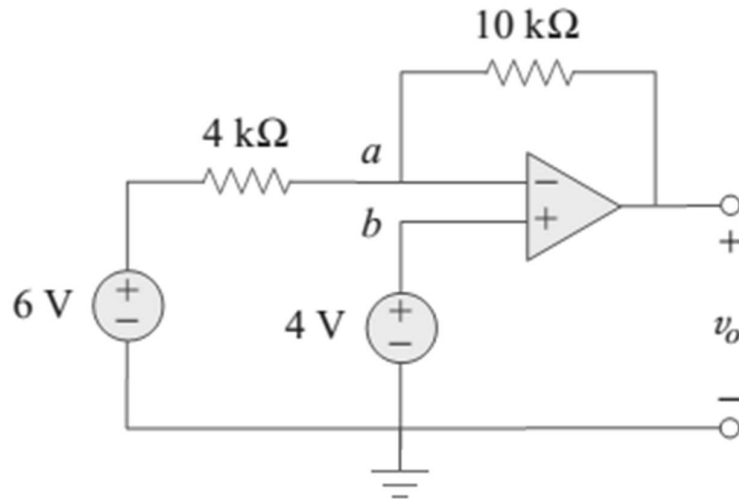
Thus amplifier with unity gain, is known as Voltage Follower, that its output follows the input.

$$v_o = v_i$$



PROBLEMS

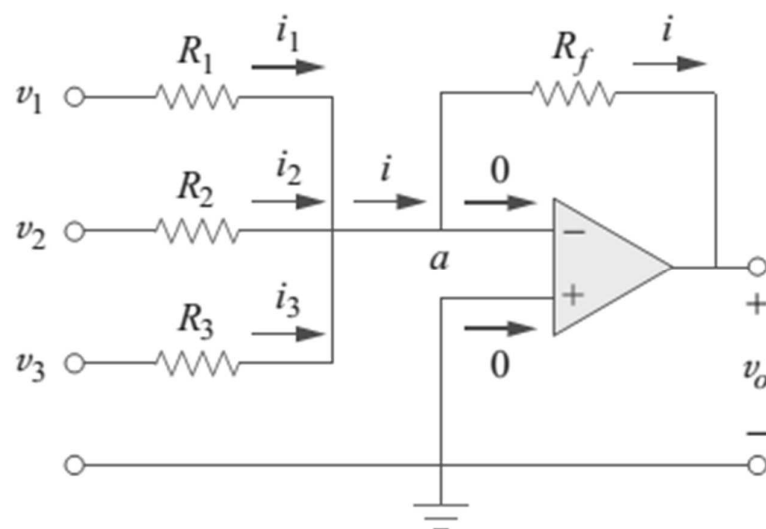
Find the output voltage?



(-1V)

SUMMING AMPLIFIER

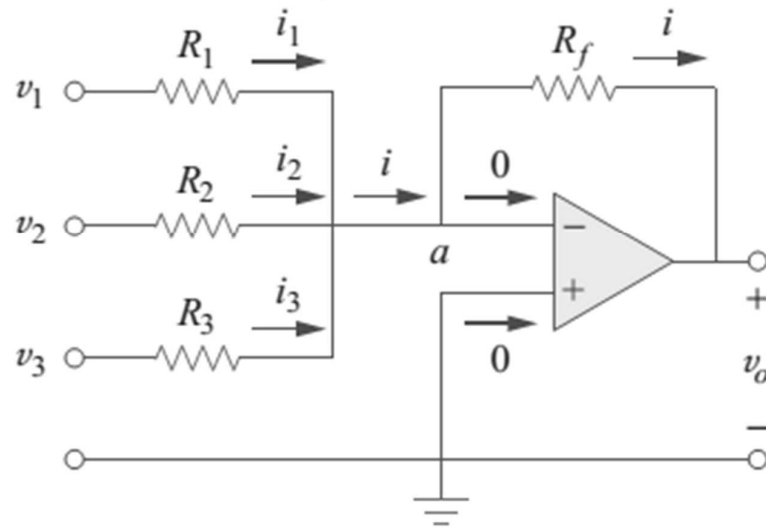
A Summing Amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.



SUMMING AMPLIFIER

The summing amplifier is a variation of the inverting amplifier.

Applying KCL at node a ; $i = i_1 + i_2 + i_3$



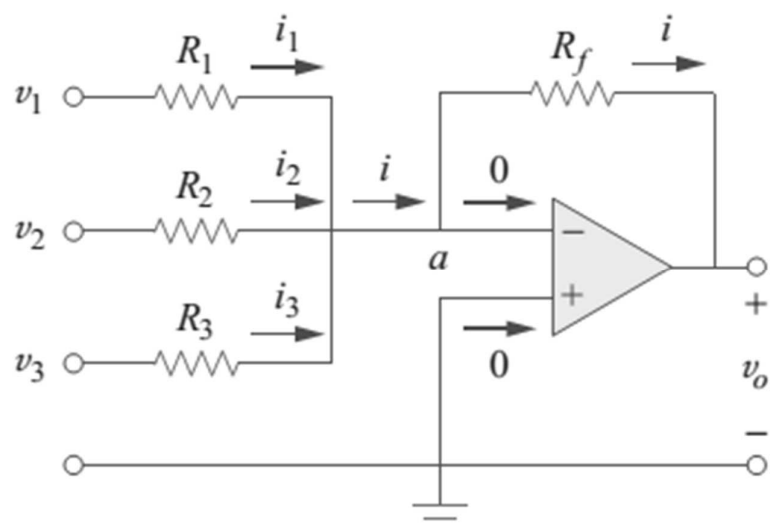
SUMMING AMPLIFIER

By Ohm's law;

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$

$$i_3 = \frac{v_3 - v_a}{R_3}$$

$$i = \frac{v_a - v_o}{R_f}$$



SUMMING AMPLIFIER

In ideal op amp; $v_a = 0$

Putting values of branch currents;

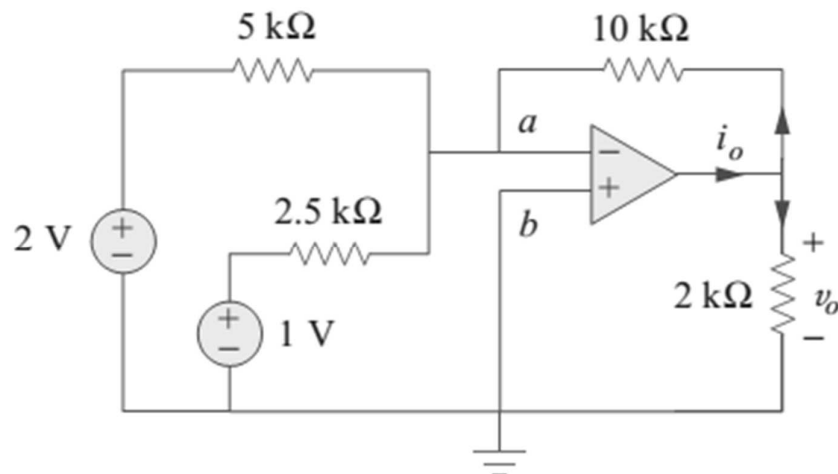
$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

The above equation indicates that output is weighted sum of the inputs, for this reason the circuit is summer.

Needless to say, the summer can have more than three inputs.

PROBLEMS

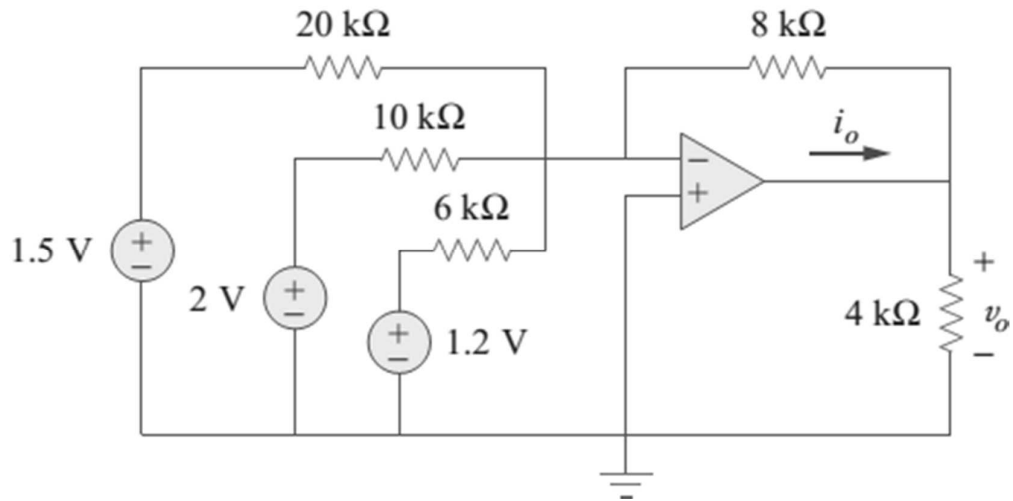
Find the v_o and i_o in op amp circuit?



(-8V, -4.8mA)

PROBLEMS

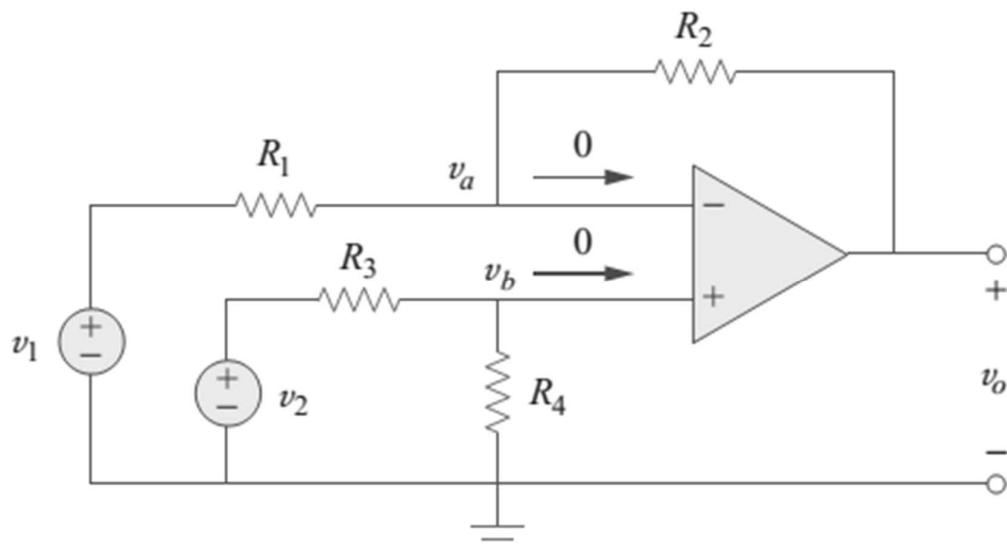
Find the v_o and i_o in op amp circuit?



(-3.8V, -1.425mA)

DIFFERENCE AMPLIFIER

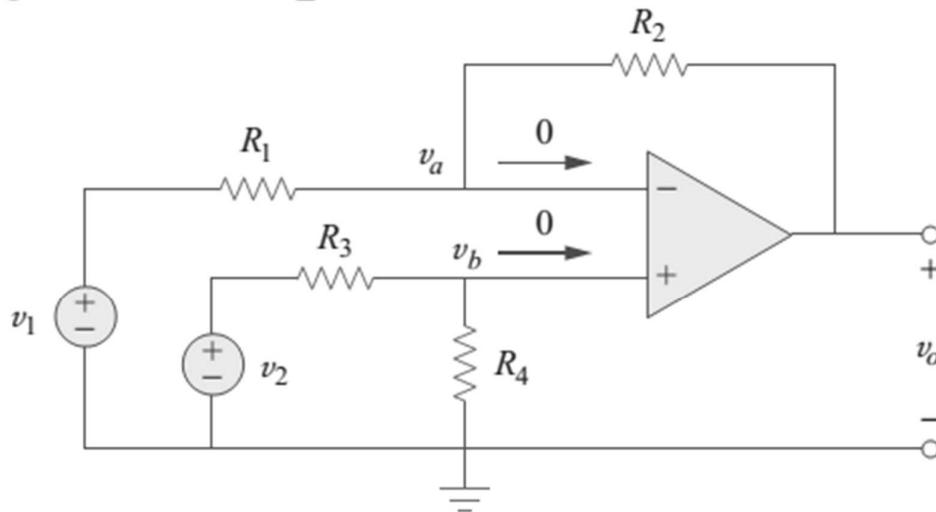
A Difference Amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.



DIFFERENCE AMPLIFIER

Applying KCL at node a ;

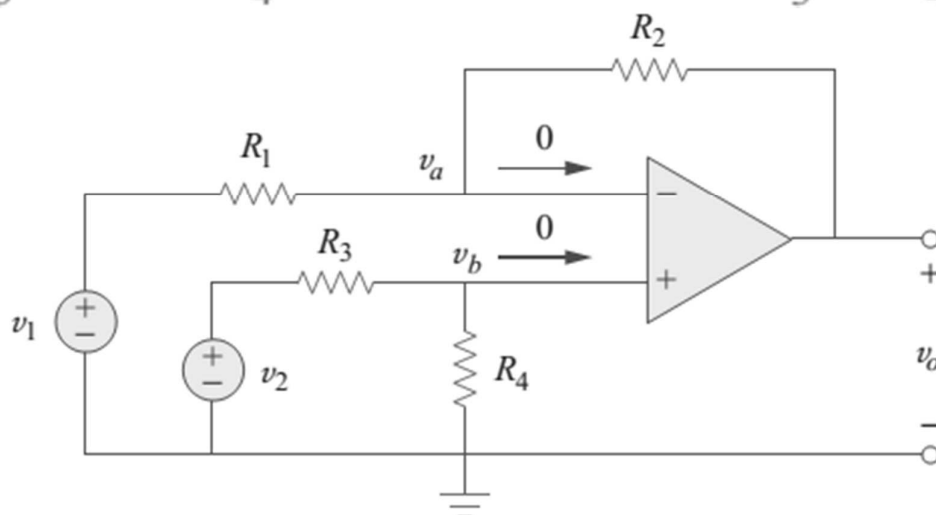
$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \quad v_o = \left(\frac{R_2}{R_1} + 1 \right) v_a - \frac{R_2}{R_1} v_1$$



DIFFERENCE AMPLIFIER

Applying KCL at node b ;

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4} \quad v_b = \frac{R_4}{R_3 + R_4} v_2$$



DIFFERENCE AMPLIFIER

Eq. at node a ;

$$v_o = \left(\frac{R_2}{R_1} + 1 \right) v_a - \frac{R_2}{R_1} v_1$$

Eq. at node b ;

$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

In ideal op amp; $v_a = v_b$

$$v_o = \left(\frac{R_2}{R_1} + 1 \right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

DIFFERENCE AMPLIFIER

Since a difference amplifier must reject a signal common to the two inputs;

$$v_o = 0 \quad v_1 = v_2$$

Then property;

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

When op amp is difference amplifier;

$$v_o = \frac{R_2}{R_1} (v_2 - v_1)$$

DIFFERENCE AMPLIFIER

If $R_1=R_2$ and $R_3=R_4$, the difference amplifier becomes a subtractor, with output;

$$v_o = v_2 - v_1$$

PROBLEMS

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$?

$$(R_2=5R_1, R_3=R_4)$$

Design a difference amplifier with gain 5?

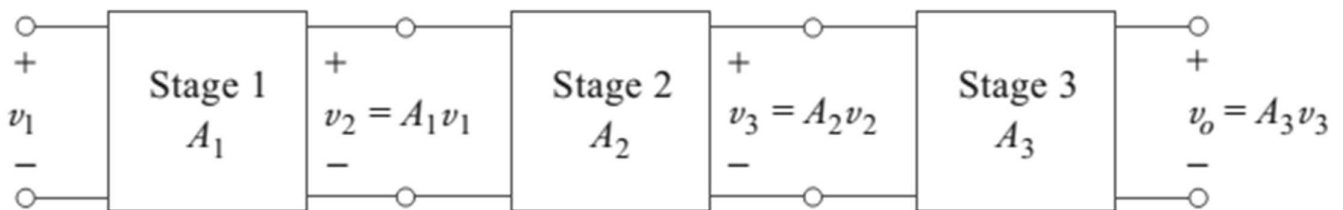
$$(R_1=R_3, R_2=R_4)$$

$$R_2=5R_1)$$

CASCADED OP AMP CIRCUITS

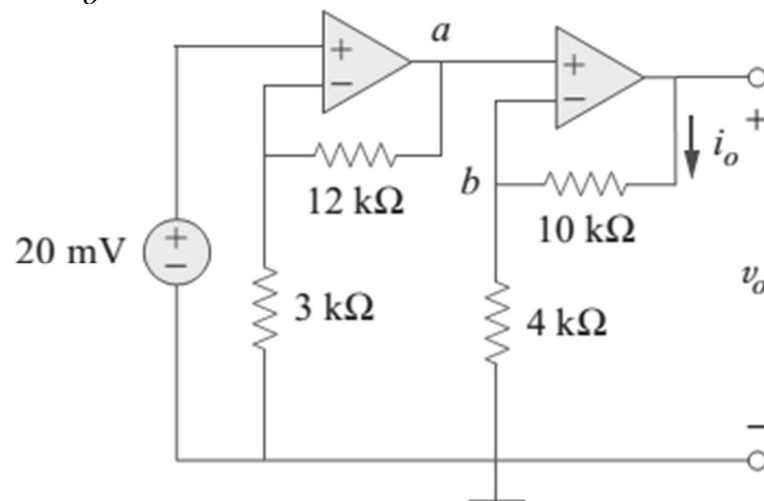
A Cascade connection is a head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next.

$$A = A_1 A_2 A_3$$



PROBLEMS

Find v_o and i_o in the circuit?



(350mV, 25μA)

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 05 – Operational Amplifiers (5.1 – 5.8)

Exercise Problems: 5.1 – 5.74

Do exercise problem yourself.